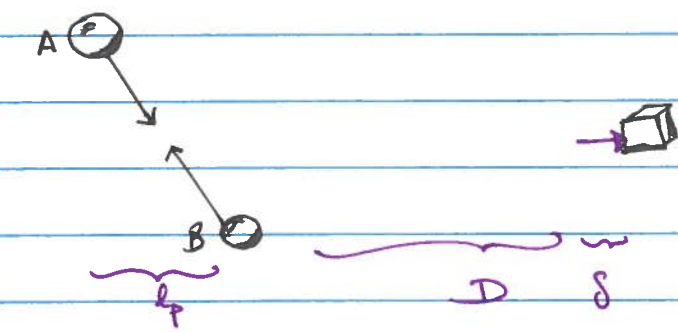
Differential Gravitational Effects — Raghu, 29 Nov. 2021



$$\Delta F_A = GmM\left(\frac{1}{(D+\ell_p)^2} - \frac{1}{(D+\ell_p+\delta)^2}\right)$$

$$\Delta F_B = GmM\left(\frac{1}{D^2} - \frac{1}{(D+\delta)^2}\right)$$

The relevant change in force is $\Delta F_B - \Delta F_A$

$$= \frac{GmM}{D^2}\left[\frac{1}{(1+\ell_p/D)^2} - \frac{1}{(1+\frac{\ell_p+\delta}{D})^2} - 1 + \frac{1}{(1+\frac{\delta}{D})^2}\right]$$

$$= \frac{GmM}{D^2}\left(-6\,\frac{\ell_p\delta}{D^2}\right)$$, keeping terms to second order in $\frac{\ell_p \text{ or } \delta}{D}$

Therefore the relevant change in force, rather than $\frac{2\delta}{D}\,\frac{GmM}{D^2}$ as I previously wrote, is

$$\text{"}\Delta F\text{"} = \boxed{\underline{\left(\frac{3\ell_p}{D}\right)}\left(\frac{2\delta}{D}\right)\frac{GmM}{D^2}}.$$

The new part, underlined, is a factor of ℓ_p/D, which isn't surprising.

With this, $\epsilon \approx 1$ at about $n=38$ rather than $n \approx 30$, or about 6 ns rather than 5. Not much of a change — our argument is barely altered. The exponential scaling w/ collisions dwarfs any prefactors!